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LIFE-CONTINGENCY TABLES.

PART I—THE CHANCES OF PREMATURE DEATH, AND THE VALUE OF SELECTION AMONG ASSURED LIVES.

BY EDWIN JAMES FARREN,

FELLOW OF THE INSTITUTE OF ACTUARIES; AND ONE OF THE ACTUARIES AUTHORIZED TO CERTIFY TABLES FOR FRIENDLY SOCIETIES.

1814

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INTRODUCTION.

THE Tables of Mortality hitherto published have been framed from Observations on *mixed* lives with regard to *health*, as in the Tables relating to the Population at large, the Returns from which have not in previous Censuses comprised the distinctions of "healthy" and "unhealthy;"—data which it is to be hoped, for the purposes of Statistical Science, will be collected at the approaching Census of 1851, as at least of equal importance with the distinctions of "occupation" and "locality" commonly included in such Returns. The Tables also, relating to the Experience of Life Assurance Societies, though framed in connection with Lives known to be in "select" health, at some former period, become also of a mixed character, if the *general* Mortality at each age alone be deduced, because persons corresponding in age may reasonably be supposed in different states of health if originally selected at different periods of life. To avoid this admixture, the primary Table of the present work has been confined to the Mortality at each age experienced by seventeen of the oldest Life Assurance Societies during the *first* Calendar year after the selection of the Lives or the year commencing with the first of January following the date of Policy.*

The author, in forming the Table from the first year's mortality already referred to, as the basis of the present work, has commenced the computation in each case from the *termination* of the "Calendar" (January to December) year of assurance, rather than from entries at different periods of that year, in order, by that procedure, to compensate for fractional ages on admission, it being a fair assumption that persons insuring at the commencement of any "Calendar" year, would complete the year of age for which the premium was exacted, *before* the termination of the "Calendar" year; while others insuring at a late period of the year, would not complete that age until *after* the year's termination; in this view, therefore, the termination of the "Calendar" year of assurance

* The experience referred to, consists of that of the Amicable and Equitable Societies, as printed under the able superintendence of their respective actuaries, and that of fifteen other Assurance Societies under the equally able superintendence of eight actuaries chiefly connected with them. The experience thus printed refers to *Lives* in the cases of the Amicable and Equitable Societies, and comprises about one-third of the total experience: the remaining data refer to *Policies*, it being almost impossible, in the experience of several offices combined, to distinguish how many Policies may have been granted upon the same Lives. It was probably at first imagined, that this want of distinction might invalidate the data as derived from Policies, but the result has shown that whether the proportions of living and dying be taken from Lives or from Policies, the *ratios* of calculation are so nearly the same when large numbers are concerned that it is immaterial which class of data be employed, the preference if any, though at first sight in favour of Lives, appearing, on consideration, rather in favour of Policies for practical purposes, Life-Assurance Societies having to discharge "claims" in respect of death not merely in relation to the number of persons deceased, but to the sums assured; which in practice are generally estimated as divisible into Policies of average amount upon the same Lives. The statistics of Policies therefore partially include an element of some utility, namely, the average sums assured, and it is probable that if the rate of Mortality at any particular age as deduced from *Lives* differed from that deduced from *Policies*, the former should be taken as representing the number of *persons* dying while the latter might be assumed as a better type of the average sums assured to be claimable in respect of such deaths. A combination, however, as in the present case, of two sets of data, the one representing Lives, and the other Policies, seems to involve all that is really desirable. The total experience alluded to, comprises 21398 Lives in relation to the Equitable Society, and 40616 Policies in relation to the combined experience of fifteen other Societies, making together 62014 assurances, capable of being traced year by year from date of Policy, to which number are to be added 16273 Policies, from the fifteen combined offices, affording data of a less precise but still useful character, and 4618 Lives from the Amicable Society somewhat differently arranged as to ages from the other data, making collectively 83905 Assurances, of which, 44877 were current at the termination of the observations, 25247 had been voluntarily discontinued, and 13781 had become claims in respect of Death. The dates of publication &c. of the Experience, will be found in a note to a subsequent page.

may upon an average be taken as the termination of the particular year of age; an assumption the more readily to be adopted, from its rendering the usual measure of half-a-year's exposure to mortality for "discontinued" assurances the more veritable, such discontinuances generally occurring at or about the time of the ensuing year's premium becoming due; and, therefore, upon an average, midway in the particular "Calendar" year of such occurrence, or six months after the completion of the last year. The calculation of such a Table as has been described, from the mortality occurring during the first year after assurance appeared desirable, not merely as the first Table of its kind, but more particularly to illustrate the chances of *premature death*, for deaths might fairly be so called which occurred among persons who a few months before had been certified to be in a sound state of health. Such a Table, moreover, might be expected to display the *extreme* value of "*selection*," and thus afford a minimum rate of mortality, a determination of some importance considered as a limit to undue diminution of premium, for no Life-Assurance Society could reasonably expect, however carefully the selection of lives might be made, to experience in after-years an annual mortality less than that experienced by other Societies in the first year after selection. A further use of such a Table would arise from its affording the true means of determining the rates chargeable for a single year's assurance at different ages, rates that could not properly be deduced from the general mortality at each age, because such mortality would relate to lives selected at various periods, and not to those only who had been recently selected.

But a still greater value would attach to such a Table, from its affording the comparative means of estimating the mortality among lives which should be rejected by Assurance Societies as not select, or only admitted upon payment of an extra premium. For having calculated a Table, showing the mortality among lives recently selected, and considered as sufficiently select for admission without extra charge, if another Table were calculated showing the annual mortality among assured lives collectively during the ensuing years of life, without new admissions, the difference between these two Tables would be typical of the difference of mortality prevailing between such of the surviving lives remaining insured, as might be *re-assured* as *select*, and the remainder who should not be so re-assured except upon payment of an extra premium. This comparison would be materially helped and verified by a third Table, representing a combination of the two former, or the general mortality at each age among assured lives, consisting not only of those admitted at earlier ages, but also including those admitted at the particular age.

A further comparison, with a Table showing the mortality experienced among the population at large, or lives wholly without selection, would lead to extra inferences as to the rates of mortality that might be expected to prevail among deteriorated lives; and thus the whole question, as to degrees of selection, might be fairly investigated.

These various reasons appeared to indicate that the calculation of three such Tables from the data alluded to would be a useful process, and the author has accordingly devoted the First Part of the present publication to the elucidation of the method adopted in framing them, and to the inferences deducible from them as to the value of selection, reserving for a subsequent part of the same work, the exemplification of their utility as collateral means in constructing Tables for the assurance of *Impaired* lives.

The consideration that there are many who are interested in the *results* obtained from vital statistics, who have neither inclination nor leisure to trace the methods of deduction employed in establishing them, has induced the author to adopt the present introduction

as preferable to any other part of the work for presenting, though by anticipation, the Tables of the rates of mortality prevailing among the two distinct classes and their combination already referred to, as compared, from their comprising chiefly male lives, with the rates of mortality in operation among the male population at large. Table A is accordingly made to represent the mortality occurring among Assured Lives during the *first* year after selection; Table B, to exhibit the general mortality in subsequent years ***without new admissions***; Table C, that of the two classes combined; and a Table, denoted by the letter P, represents the general mortality prevailing among the male population of England and Wales without selection, deduced for the purposes of this comparison, from the excellent Table of Mortality by Mr. Farr, inserted in the Appendix to the Sixth Annual Report of the Registrar General. A comparison of each of the others with Table P, affords the means of deducing the necessary inferences.

Comparison of TABLES A and P.

TABLE A indicates the Rates of Mortality occurring among Assured Lives during the *First* year after Selection, or the Chances of Premature Death.

TABLE P indicates the Rates of Mortality occurring among the Male Population.

A

Age.	Year's Mortality ℘ Cent. First Year after Selection.	Year's Mortality ℘ Thousand at Ten Ages.
21	·5891	
22	·5965	
23	·6036	
24	·6106	
25	·6175	
26	·6247	
27	·6321	
28	·6401	
29	·6488	
30	·6584	6·2214
31	·6692	
32	·6813	
33	·6951	
34	·7107	
35	·7284	
36	·7485	
37	·7713	
38	·7971	
39	·8262	
40	·8588	7·4866
41	·8954	
42	·9362	
43	·9816	
44	1·0320	
45	1·0877	
46	1·1491	
47	1·2166	
48	1·2906	
49	1·3714	
50	1·4595	11·4201
51	1·5553	
52	1·6593	
53	1·7718	
54	1·8933	
55	2·0243	
56	2·1651	
57	2·3164	
58	2·4784	
59	2·6517	
60	2·8368	21·3524
61	3·0341	
62	3·2441	
63	3·4673	
64	3·7041	
65	3·9551	
66	4·2206	
67	4·5013	
68	4·7974	
69	5·1096	
70	5·4382	41·4718

P

Age.	Year's Mortality ℘ Cent. Male Population, without Selection.	Year's Mortality ℘ Thousand, at Ten Ages.
21	·8122	
22	·8331	
23	·8547	
24	·8767	
25	·8994	
26	·9227	
27	·9466	
28	·9711	
29	·9962	
30	1·0220	9·1347
31	1·0485	
32	1·0757	
33	1·1035	
34	1·1321	
35	1·1614	
36	1·1915	
37	1·2223	
38	1·2539	
39	1·2863	
40	1·3195	11·7947
41	1·3536	
42	1·3884	
43	1·4242	
44	1·4608	
45	1·4982	
46	1·5366	
47	1·5759	
48	1·6162	
49	1·6573	
50	1·6994	15·2106
51	1·7426	
52	1·7866	
53	1·8317	
54	1·8779	
55	2·1069	
56	2·3327	
57	2·5092	
58	2·7030	
59	2·9145	
60	3·1441	22·9492
61	3·3927	
62	3·6612	
63	3·9507	
64	4·2625	
65	4·5983	
66	4·9596	
67	5·3483	
68	5·7665	
69	6·2263	
70	6·7001	48·8662

From the comparison of these two Tables, it appears that although "Selection" has a decided influence in reducing the ensuing year's mortality, yet that premature deaths, even among lives recently selected, must be expected to occur at all ages, and to be irreducible below a certain minimum whenever large numbers are concerned. The value of selection during the first year has been to ward off from the Life Assurance Societies during that year about the following number of additional "claims" that might have been expected to occur if the Assurances had been issued upon lives among the male population at large, without discrimination as to health.

Among 1000 new assurances granted						Between ages 21	and	30,	about	3	claims, (9,1347—6,2214.)
,,	,,	,,	,,			31	,,	40	,,	4	,, (11,7947—7,4866.)
,,	,,	,,	,,			41	,,	50	,,	4	,, (15,2106—11,4201.)
,,	,,	,,	,,			51	,,	60	,,	2	,, (22,9492—21,3524.)
,,	,,	,,	,,			61	,,	70	,,	7	,, (48,8662—41,4718.)
5000		,,	,,			21	,,	70	,,	20	,, (107,9554—87,9523.)

or the claims were less to the extent stated, or 20 per 5000 assurances during the first year after selection, than if such had been issued at the rate of 1000 to each decennial of age among the male population indiscriminately, supposing that a fair average of such lives could have been secured.

It appears from the foregoing summary that during the decennial 51 to 60, or the "break of life," as it has been termed, the value of selection is at its minimum, an indication that may be accepted in its general form, though not so entirely as to rely upon the subdivisions of mortality of Table P at such ages, for the principle of construction adopted by Mr. Farr in the Table of Mortality from which it is deduced, being virtually that of subdivision into two distinct parts, or of arranging the mortalities into two geometrical progressions, the years of junction, (in the present instance, from 51 to 60,) have too specific an effect to be regarded otherwise than as a necessity arising from the principle of calculation adopted. The comparative decrease, however, in the value of selection during this decennial, is of too obvious a character to be wholly explained as a mere result of calculation; it is therefore probable that persons seeking Life Assurance between these ages, form a better criterion of their own health than at other periods of life, and thus render the task of selection by parties having an opposite interest, more than usually difficult.

During the succeeding decennial of 61 to 70, the value of selection, it will be seen, regains its power, and probably retains it to the more advanced ages, though the number of lives admitted after 70 are too few to enable us to tabulate the results.

Table A, and the inferences deducible from it being thus explained, the elucidation of Table B, or that representing the mortality in after-years, *without new admissions*, next demands attention.

TABLE B, illustrating the Yearly Mortality experienced at all ages collectively, according to the number of "Calendar" years that had elapsed from Dec. 31, after Admission of the Lives as in "Select" health.

Column I.—Number of "Calendar" Years from Dec. 31, after Admission.

II.—Number of *Assurances comprised* during following year

III.— „ „ *Assurances discontinued* „ „ „ except by reason of Death.

IV.— „ „ *Assurances current* „ „ „ at termination of Observations.

V.— „ „ *Assurances becoming Claims* „ „ by reason of Death.

VI.— „ „ *Assurances not passing on* „ „ or Sum of III, IV, and V.

VII.— „ „ Assurances passing on to next year, *doubled* to avoid half years for "discontinued" and "current" Assurances, or double the difference between II and VI.

VIII.—Representative number exposed to full year's risk of mortality, doubled to avoid, &c. or Sum of V, VI, and VII, increased by "Existing Assurances" of the Equitable Society's experience.

IX.—Representative number of Deaths, doubled to preserve relation to VIII.

X—Relative Mortality during the Year, *per Thousand.*

I. Years elapsed from Dec. 31, after Admission.	II. Assurances comprised.	III. Assurances Discontinued.	IV. Assurances Current.	V. Assurances becoming Claims.	VI. Total not passing on.	VII. 2(II-VI).	VIII. V+VI+VII +Eq: IV.	IX. 2(V).	X. Mortality ₱ 1000.
	62014	204	2894	177	3275	.....	.....	....	.....
Dec. 31. 0	58739	3647	2553	576	6776	103926	111278	1152	10·352
„ „ 1	51963	3088	2650	730	6468	90990	98360	1460	14·843
„ „ 2	45495	3099	2422	669	6190	78610	85652	1338	15·621
„ „ 3	39305	2013	2036	565	4614	69382	74738	1130	15·120
„ „ 4	34691	1632	2011	559	4202	60978	65932	1118	16·957
„ „ 5	30489	1205	1676	524	3405	54168	58273	1048	17·984
„ „ 6	27084	1597	1653	487	3737	46694	51104	974	19·059
„ „ 7	23347	914	1466	418	2798	41098	44508	836	18·783
„ „ 8	20549	816	1427	364	2607	35884	39090	728	18·624
„ „ 9	17942	400	1422	343	2165	31554	34306	686	19·997
„ „ 10	15777	316	1205	323	1844	27866	30258	646	21·350
„ „ 11	13933	255	1534	309	2098	23670	26306	618	23·493
„ „ 12	11835	229	1402	245	1876	19918	22293	490	21·980
„ „ 13	9959	162	1032	244	1438	17042	19042	488	25·628
„ „ 14	8521	155	719	201	1075	14892	16474	402	24·402
„ „ 15	7446	147	479	200	826	13240	14565	400	27·463
„ „ 16	6620	100	431	168	699	11842	13021	336	25·805
„ „ 17	5921	66	322	153	541	10760	11731	306	26·085
„ „ 18	5380	70	311	152	533	9694	10664	304	28·507
„ „ 19	4847	69	358	137	564	8566	9613	274	28·399
„ „ 20	4283	53	268	140	461	7644	8493	280	32·968
„ „ 21	3822	48	241	127	416	6812	7580	254	33·377
„ „ 22	3406	38	221	127	386	6040	6750	254	37·630
„ „ 23	3020	29	186	116	331	5378	6005	232	38·635
„ „ 24	2689	34	167	102	303	4772	5344	204	38·174
„ „ 25	2386	19	149	95	263	4246	4751	190	39·992

EXAMPLE.—The number of Assurances comprised at periods of admission was 62014, of these 204 were voluntarily discontinued on or before Dec. 31;—2894 were still current as New Policies at termination of the Observations, and 177 became claims in respect of Death before the close of the "Calendar" year of Assurance, the total therefore not passing on, was 3275, which subtracted from 62014, left 58739 to be comprised by the following year. Of these, 3647 were voluntarily discontinued on or before Dec. 31;—2553 were current in ensuing Calendar year after that of admission at termination of the observations, and 576 became Claims in respect of Death during the same year; the total, therefore, not passing on, was 6776, which subtracted from 58739, left 51963, to be comprised by the following year.—The resulting mortality during the ensuing "Calendar" year after that of admission, it will be seen by Column X, was 10·352 per Thousand. The remainder of the Table is framed on similar principles.

To compare the results in Table B with those in Table P, it will be necessary to refer to Col. X, and by the mortality there expressed, to ascertain the age at or about which such mortality would have occurred among the Male population, without descrimination as to health, and from the age so obtained the relative age at which the male population might have been admitted without selection as to health, to experience the same mortality as among assured Lives during any particular year after admission, is easily obtained by aid of Col. I.—For example, the ensuing year's mortality experienced by Assured Lives after that of admission, is by Col. X, at the rate of 10·352 per thousand, or 1·0352 per cent, which on reference to Table P., will be found to be about the mortality experienced by the male population at or about the age of 31, consequently Lives at or about the age of 31, might have been admitted without descrimination from the Male population, to experience the same collective mortality as that experienced at all ages among assured Lives, during the calendar year following that of admission. In the same manner, the mortality after one year from that of admission among assured Lives, is found by Col. X., to be 14·843 per thousand, or 1·4843 per cent., which on reference to Table P., may be seen to be that at or about the age of 45 among the Male population, consequently the relative age on admission among such Lives would have been one year earlier, or about the age of 44, to experience one year after admission the same rate of mortality as that experienced by select Lives of the same standing. As a last example, it will be seen in Col. X. that 25 years after that of admission, assured Lives experience a rate of mortality equalling 39·992 per thousand, or 3·9992 per cent., which in Table P. is nearest to that of age 63, consequently 25 years previously Lives from among the Male population might have been admitted at or about the age of 38, to experience 25 years after admission the same mortality as among assured Lives of equal standing.

The following Comparative Table is formed in the preceding manner, Cols. b and e being taken from Table B, and Columns a, c, and d, added by comparison with Table P.

a	b	c	d	e
Nearest Relative ages on admission among the Male population to experience, during particular years after admission about the same mortality as among the Assured Lives.	Additional Age, or Years elapsed from Dec. 31 after admission.	Corresponding Ages attained at particular years after admission.	Mortality ℘ Cent. among Male Population following particular years after admission. Table P.	Mortality ℘ Cent. among assured Lives following particular years after admission. Table B.
	Dec. 31. 0	31	1·0485	1·0352
44	,, 1	45	1·4982	1·4843
45	,, 2	47	1·5759	1·5621
43	,, 3	46	1·5366	1·5120
46	,, 4	50	1·6994	1·6957
47	,, 5	52	1·7866	1·7984
48	,, 6	54	1·8779	1·9059
47	,, 7	54	1·8779	1·8783
46	,, 8	54	1 8779	1·8624
46	,, 9	55	2·1069	1·9997
45	,, 10	55	2·1069	2·1350
45	,, 11	56	2·3327	2·3493
43	,, 12	55	2·1069	2·1980
44	,, 13	57	2·5092	2·5628
43	,, 14	57	2·5092	2·4402
43	,, 15	58	2·7030	2·7463
41	,, 16	57	2·5092	2·5805
41	,, 17	58	2·7030	2·6085
41	,, 18	59	2·9145	2·8507
40	,, 19	59	2·9145	2·8399
41	,, 20	61	3·3927	3·2968
40	,, 21	61	3·3927	3·3377
40	,, 22	62	3·6612	3·7630
40	,, 23	63	3·9507	3·8635
39	,, 24	63	3·9507	3·8174
38	,, 25	63	3·9507	3·9992

EXAMPLE.—Population Lives admitted at age 44 would, one year after admission, or at the age of 45, experience a mortality of 1·4982 per cent.; while Assured Lives one year after admission, experience at all ages collectively a mortality of 1·4843 per cent.

An examination of column a, shows that the relative ages on admission among the Male population, to subsequently experience about the same rates of mortality as among the assured survivors of originally Select Lives, would range from 38 to 48 a decennial as closely indicating the real average ages among the assured Lives, as any other that can be named, considering, in accordance with this column, that a large proportion of the greater ages, would continue assured to about six years duration, the discontinuances being principally among the younger lives; but that afterwards the reverse would occur by the increased mortality among the more aged. The proximity, therefore, of the results derived from Tables B and P, leads us to the conclusion that any reduced rate of mortality among assured lives collectively, must be mainly dependent on Table A, or that representing the mortality among the new lives, which by admixture with those already assured tend to reduce the relative mortality; but that without this admixture the rates of mortality among the survivors remaining assured would not particularly differ from those prevailing among the male population at large, taken indiscriminately without regard to health. This conclusion is fully warranted by the actual experience of the Equitable Society, which has been found to indicate about the same average duration of Life as the collective Experience of the other Societies. The Experience of the Equitable Society alluded to, denotes that Lives admitted during the decennial preceding the age of 35, subsequently experience on attaining that age, an average duration

of 29·10 years, while the average duration among the Male population at that age, is 29·83. The following Table shows the relative values on the same principle, at the ages of 35, 45, 55, and 65. The first set of values is obtained from Tables, H. 1, 2, 3, and 4, of the Official Experience already referred to, as printed under the superintendence of a committee of actuaries, the second relates to Table P, as inserted at p. xix of the Registrar General's Fifth Report.

Ages following each decennial of admission.	Equitable Society. — Average duration.	Male population. — Average duration.	difference.
35	29·10	29·83	— 0·73
45	23·69	23·30	+ 0·39
55	17·03	16·68	+ 0·35
65	11·08	10·86	+ 0·22

Considering the certificates of health, and other tests applied by Assurance Companies, it may at first sight appear somewhat anomalous to those unaccustomed to such details, that the mortality experienced by the Equitable and other Societies, professedly accepting none but select Lives, should be nearly as great in after-years of assurance, as that displayed among the Male population indiscriminately, a result fairly deducible as has been shewn from Table B. This result, however, becomes the more trustworthy, from its being corroborated by the opinion of one who has rarely, if ever, delivered a false judgment on such matters, the authority here alluded to, being the eminent actuary, Mr. Milne, constructor of the well-known "Carlisle Table of Mortality." In an article on Human Mortality in the present edition of the Encyclopædia Britannica, p. 555, this celebrated author thus writes:—

"Although the Members of such a Society when they first enter, are Select Lives, they are not even then so much better than the common average as many persons suppose; for the more precarious a Life is, the stronger is the inducement for parties interested in its continuance to get it insured, so that bad risks are frequently offered to such Companies; and many proposals for Insurance are accepted by the Directors that are not thought very eligible at the time, in cases where they are not aware of any specific objection to the Life proposed. Besides, it is to be considered that of the number in a Society at any one time, but a small proportion can have been recently admitted, and *in a few years from the time of admission the members will generally have come down to the common average of persons of the same ages.*"

In the same publication, Art. Annuities, p. 203, Mr. Milne also observes,—"There is reason to believe that all the caution in selection, which the offices in general can exercise, is necessary to keep the Lives insured up to the average goodness of the bulk of the population."

The whole doctrine of mathematical probability being founded upon the law of average, for it is only upon an average of results that the indications of probability can be verified, or even fairly deduced, a Table that can be thus relied on as a *general* average Table, both of time and occurrence, is of inestimable value in vital statistics, for by its very generality it not only affords a central line between extreme classes, but as in the case of Table P between those classes, where the differences would be supposed more minute, as among those constituting Table B.

From the result established by Table A, or that of the first year's Mortality being less than that represented by Table P, and from the conclusion deducible from Table B that the successive Mortalities of after-years are closely allied to those detailed in Table P, it should follow that a Table representing the combined Mortality or that arising from the admixture of former with newly-admitted Lives, should be greater than that of Table A, from including former admissions, and less than Table B or its equivalent Table P by including recent admissions. All these indications will be found fulfilled by Table C which is constructed from the total data or 83905 Assurances, as preferable to any less number, and is here placed midway between Tables A and P, for the purposes of comparison.

Table C, and others similarly constituted, in addition to their general value have also the collateral one referred to, in an early part of the present Introduction, of aiding the determination in connection with Tables A and B, of the degrees of deterioration of health, through which originally select lives eventually pass. An illustration of this occurs in the proportionately great mortality at age 21, in Table C, as compared with that in Table A, at the same age, a difference that is thus indicated to arise from the inferior value of selection as exercised on the lives of minors, about two-thirds of the lives at that age having been selected previous to the age of majority.

The Tables A, C, and P, thus compared, denote the value of selection year by year. To display the collective value, the succeeding Tables, AA, CC, have been calculated, showing the average duration or expectation of life, in relation to Tables A and C; with an Additional Table BP, in similar relation to Table P, or has been shown, pp. viii to xi, to Table B. This form of comparison is adopted, because it not only shows the average duration, but admits also of being considered as showing the present value of an annuity of £1 when interest of money is disregarded, and half-a-year's payment is claimable in respect of the year in which death shall happen.

It now only remains to recapitulate succinctly the deductions of the present introduction, before passing on to the succeeding section, in which the methods of calculation employed in constructing the Tables from the assigned data are explained. The three deductions exemplified by the Tables A, AA; B, BP; and C, CC; are as follows:

From Tables A, AA.—The value of selection is operative at all ages in reducing the first year's mortality; but is least so during the decennial of age 51 to 60. It is not sufficiently potent at any age to wholly abolish the chances of premature death; an assignable rate of mortality prevailing at all ages even after recent selection.

From Tables B, BP.—The value of selection is scarcely traceable after the first years, being nearly wholly neutralized by attending causes, particularly by discontinuances on the part of healthy lives, soon after selection. The deterioration ultimately merges into about the same rates of mortality as those occurring among the Male population.

From Tables C, CC.—New admissions tend to retard the increase of mortality, otherwise arising from deterioration of the health of assured survivors, the joint mortality at the majority of ages being less than that among the male population at large, but closely approximating to it during the decennial 51 to 60.

TABLE A.—Showing the Rates of Mortality occurring amongst Assured Lives during the First Year after Selection, or the chances of Premature Death.

Age.	Mortality ℘ Cent.	Mortality ℘ Thousand at Ten Ages.
21	·5891	
22	·5965	
23	·6036	
24	·6106	
25	·6175	
26	·6247	
27	·6321	
28	·6401	
29	·6488	
30	·6584	6·2214
31	·6692	
32	·6813	
33	·6951	
34	·7107	
35	·7284	
36	·7485	
37	·7713	
38	·7971	
39	·8262	
40	·8588	7·4866
41	·8954	
42	·9362	
43	·9816	
44	1·0320	
45	1·0877	
46	1·1491	
47	1·2166	
48	1·2906	
49	1·3714	
50	1·4595	11·4201
51	1·5553	
52	1·6593	
53	1·7718	
54	1·8933	
55	2·0243	
56	2·1651	
57	2·3164	
58	2·4748	
59	2·6517	
60	2·8368	21·3524
61	3·0341	
62	3·2441	
63	3·4673	
64	3·7041	
65	3·9551	
66	4·2206	
67	4·5013	
68	4·7974	
69	5·1096	
70	5·4382	41·4718

TABLE C.—Showing the General Rates of Mortality occurring amongst Assured Lives, including both former and recent Admissions.

Age.	Mortality ℘ Cent.	Mortality ℘ Thousand, at Ten Ages.
21	·7577	
22	·7691	
23	·7810	
24	·7932	
25	·8057	
26	·8186	
27	·8317	
28	·8451	
29	·8589	
30	·8732	8·1342
31	·8879	
32	·9035	
33	·9199	
34	·9373	
35	·9561	
36	·9764	
37	·9986	
38	1·0229	
39	1·0498	
40	1·0796	9·7320
41	1·1127	
42	1·1494	
43	1·1905	
44	1·2362	
45	1·2871	
46	1·3437	
47	1·4068	
48	1·4767	
49	1·5542	
50	1·6398	13·3971
51	1·7344	
52	1·8386	
53	1·9530	
54	2·0785	
55	2·2159	
56	2·3659	
57	2·5292	
58	2·7069	
59	2·8996	
60	3·1082	23·4302
61	3·3337	
62	3·5769	
63	3·8338	
64	4·1200	
65	4·4216	
66	4·7447	
67	5·0899	
68	5·4581	
69	5·8504	
70	6·2676	46·7017

TABLE P.—Showing the Rates of Mortality occurring amongst the Male Population, or those among Assured Lives, as deduced from Table B, pp. viii to xi, excluding recent admissions.

Age.	Mortality ℘ Cent.	Mortality ℘ Thousand at Ten Ages.
21	·8122	
22	·8331	
23	·8547	
24	·8767	
25	·8994	
26	·9227	
27	·9466	
28	·9711	
29	·9962	
30	1·0220	9·1347
31	1·0485	
32	1·0757	
33	1·1035	
34	1·1321	
35	1·1614	
36	1·1915	
37	1·2223	
38	1·2539	
39	1·2863	
40	1·3195	11·7947
41	1·3536	
42	1·3884	
43	1·4242	
44	1·4608	
45	1·4982	
46	1·5366	
47	1·5759	
48	1·6162	
49	1·6573	
50	1·6994	15·2106
51	1·7426	
52	1·7866	
53	1·8317	
54	1·8779	
55	2·1069	
56	2·3327	
57	2·5092	
58	2·7030	
59	2·9145	
60	3·1441	22·9492
61	3·3927	
62	3·6612	
63	3·9507	
64	4·2625	
65	4·5983	
66	4·9596	
67	5·3483	
68	5·7665	
69	6·2163	
70	6·7001	48·8562

Table AA,—indicates the present value of an Annuity of £1 dependent on a Life *annually selected* from age to age, from Assured Lives on admission; in connection with Table A.

„ CC,—indicates the present value of an Annuity of £1 dependent on a Life *annually nominated by lot* from age to age, from Assured Lives, including both former and recent admissions; in connection with Table C.

„ BP,—indicates the present value of an Annuity of £1 dependent on a Life *annually nominated by lot* from age to age, from Assured Lives excluding recent admissions, or from the Male Population; in connection with Tables B and P, pp. viii to xi.

The value of the Annuity, in each case, by being estimated without consideration of interest of money, and on the supposition of half a year's payment being claimable in respect of the year in which death shall happen, corresponds to the Average duration or Expectation of Life.

AA.

Age at commencement.	Assured Lives. — Annual Nomination by Selection.
21	43·53
22	42·78
23	42·04
24	41·29
25	40·54
26	39·79
27	39·03
28	38·28
29	37·52
30	36·77
31	36·01
32	35·24
33	34·48
34	33·72
35	32·96
36	32·20
37	31·44
38	30·68
39	29·92
40	29·16
41	28·41
42	27·66
43	26·92
44	26·18
45	25·45
46	24·73
47	24·01
48	23·30
49	22·59
50	21·90
51	21·22
52	20·55
53	19·88
54	19·23
55	18·60
56	17·97
57	17·36
58	16·76
59	16·17
60	15·60
61	15·04
62	14·49
63	13·96
64	13·44
65	12·94
66	12·45
67	11·98
68	11·52
69	11·08
70	10·65

CC.

Age at commencement.	Assured Lives. — Annual Nomination by Lot.
21	40·47
22	39·78
23	39·08
24	38·39
25	37·69
26	36·99
27	36·29
28	35·59
29	34·89
30	34·19
31	33·49
32	32·78
33	32·08
34	31·37
35	30·66
36	29·95
37	29·25
38	28·54
39	27·82
40	27·11
41	26·41
42	25·70
43	24·99
44	24·28
45	23·58
46	22·88
47	22·19
48	21·50
49	20·81
50	20·13
51	19·46
52	18·80
53	18·14
54	17·49
55	16·85
56	16·22
57	15·60
58	14·99
59	14·40
60	13·81
61	13·24
62	12·68
63	12·13
64	11·59
65	11·07
66	10·56
67	10·06
68	9·57
69	9·10
70	8·63

BP.

Age at commencement.	Male Population. — Annual Nomination by Lot.
21	39·19
22	38·51
23	37·83
24	37·15
25	36·47
26	35·80
27	35·13
28	34·46
29	33·79
30	33·13
31	32·47
32	31·80
33	31·14
34	30·49
35	29·83
36	29·17
37	28·52
38	27·87
39	27·21
40	26·56
41	25·91
42	25·26
43	24·61
44	23·96
45	23·30
46	22·65
47	22·00
48	21·34
49	20·68
50	20·02
51	19·36
52	18·70
53	18·03
54	17·36
55	16·68
56	16·03
57	15·40
58	14·78
59	14·18
60	13·59
61	13·01
62	12·45
63	11·91
64	11·38
65	10·86
66	10·36
67	9·87
68	9·40
69	8·95
70	8·51

EXAMPLE,—An annuity dependent on a Life nominated, *by selection*, from 21 to 22, with re-selection from 22 to 23, 23 to 24, &c. has a total value of 43·53; whereas at age 21, an annuity dependent on an Assured Life, also nominated annually, but by *lot*, has a value of 40·47; and an annuity at age 21 on a Life annually nominated by *lot* from Assured Lives excluding recent admissions, or from Male Population, has a value of 39·19.

The chances, of the selection being well or badly exercised from year to year, may be denoted by the relative proportions surviving and dying, as indicated by the mortality per cent. of previous Table.

END OF INTRODUCTION.

SECTION I.

ON THE METHODS TO BE OBSERVED IN FRAMING TABLES OF MORTALITY FROM ASSIGNED DATA.

By aid of the processes of the Differential and Integral Calculus, it can be ascertained that if the difference of two Napierian logarithms be taken as the limit to the sum of a series, that series may be taken of the form

$$\frac{d}{l} + \frac{{}^1d}{l\text{-}d} + \frac{{}^2d}{l\text{-}(d+{}^1d)} + \cdots\cdots\cdots\cdots + \frac{{}^{x\text{-}1}d}{{}^{x\text{-}1}l}$$

in which l and $l-(d+{}^1d+\cdots\cdots\cdots+{}^{x\text{-}1}d)={}^{x\text{-}1}l+{}^{x\text{-}1}d={}^xl$ represent the numbers to which the Napierian logarithms are taken; $d,{}^1d,{}^2d\ldots{}^{x\text{-}1}d$ are supposed equal and the smallest possible; or otherwise following any law, which upon the smallest possible being represented by zero, involves the others being represented by the same symbol.

Putting λ for common logarithm, and μ for the modulus of the system, the above theorem in the usual notation of the calculus may be expressed by

$$\mu^{-1}\lambda l - \mu^{-1}\lambda\, {}^xl = -\int_l^{{}^xl}\frac{1}{l}dl = \int_{{}^xl}^{l}\frac{1}{l}dl$$

Now the above series is one in the precise form of that, which would be eventually, afforded by a Table of Mortality, minutely subdivided into ages, if the numerators were made the decrements, and the denominators the number of survivors at the successive ages commencing with that in connection with l. To give a *verbal* interpretation to such an application of the series, the definition would be, the proportion of deaths occurring among a community kept *uniform in number*, by the admission of new members *pari passu* with the deaths; a state nearly obtaining among the Population at large for short periods, the daily births and progression of ages so nearly acting as counterbalances to the vacancies occasioned by death, that two Censuses following closely upon each other might be expected to yield about the same relative results.

A rate of mortality so deduced, may be termed the *integral* rate, in relation to its being derived from the integration of $-\frac{1}{l}\,dl$; and for the purposes of distinction, the rate of mortality usually deduced and expressing the difference between the proportion of survivors and the number originally exposed to mortality may be termed the *differential* rate. The distinction between these two rates, obviously is, that the former expresses the power of mortality acting upon a uniform number exposed throughout, while in the latter case, the power of mortality merely acts from time to time upon a diminishing number of survivors, and thus, at the close of any period, involves numbers less than those at commencement; a relation that renders the former more homogeneous for *progressional* comparisons at all ages, but more especially at those relating to advanced periods of life.

The measure of the *integral* rate of mortality has been shown above to be indicated by the difference between the Napierian logarithms of the numbers living at commencement of and surviving any period, whence its relation to the *differential* rate, and to the medium number of survivors to be considered as constantly exposed during the period, is easily deduced. But a more valuable relation for the purposes of calculation consists, in the integral rate of mortality merely differing in algebraical sign from the *Napierian loga-*

rithm of the probability of surviving. For the Napierian logarithm of such probability is the negative difference between the same two Napierian logarithms as determine the *integral* rate of mortality; consequently if ${}_xl$ be made to represent the probability of living x years (or periods) at any particular age, $\mu^{-1}\lambda_x l$ will represent the Napierian logarithm of such probability; and that of its reciprocal or $\mu^{-1}\lambda_x l^{-1}$ the *integral* rate of mortality during the same time. Consequently, whatever progression prevails among the integral rates of mortality at different ages, the same progression will be found to prevail among the logarithms of the probabilities of living, and vice versa.

Having thus deduced that the two values referred to, differ only in sign, the integral rate of mortality being affected by the sign plus, and the Napierian logarithm of the probability of surviving, by that of minus, it will be convenient to henceforth treat them as of the same *numerical* value, presuming the distinction of sign to have been already sufficiently indicated to prevent misapprehension when the signs are not specifically noted. Putting then $\int m$ to denote the integral rate of mortality at any particular age, and during any particular year or period under remark, and remembering that if $l=1$, or unity be taken as the representative number alive at the commencement of any year or period, then ${}^x l$ will equal ${}_x l$, or the proportion surviving also indicates the probability of living x years or periods, we obtain the following equations, taking a single year or period, in the first instance, as the simplest form, or $x=1$

$$\mu^{-1}\lambda^1 l = \int m, \therefore {}^1 l = \epsilon^{\int m}$$

in which ϵ expresses 2·7182818, or the base of the Napierian or hyperbolic system of logarithms.

If the integral rate of mortality were presumed to be equal at all ages, it is obvious the probabilities of surviving any defined period would also be equal at all ages, or the number of survivors would decrease in geometrical progression; an equal proportion of the survivors dying in each successive year; thus the number surviving x years would be ${}^x l = ({}^1 l)^x = \epsilon^{x\int m}$; or the sum of the integral rates of mortality in x years, would be merely x times the integral rate of the first year. It is now, however, generally admitted by Actuaries, that the rates of mortality after infancy and consequently the premiums of Assurance, should be graduated so as to increase with the age, although in nearly all original data, deviations from this presumed increase in the rate of mortality have been observable. These deviations, however, not being of a fixed and determinate character, but differing in different data, have been gradually disregarded in practice, and a conditional augmentation generally adopted. To include this condition of a progressive increase in the rate of mortality, it is obvious that whatever may be the nature of the original data at particular ages, it will be necessary in a practicable table of mortality that the representative number of *survivors* should decrease in a greater ratio than that of geometrical progression, which supposes an equable rate of mortality, as shown by the preceding equation, ${}^x l = ({}^1 l)^x = \epsilon^{x\int m}$.

One of the most ingenious assumptions to accomplish the purpose of augmentation, we owe to the able mathematician, Mr. Gompertz, who, in the Philosophical Transactions of 1825, Part II, p. 517, in continuation of a former paper, (1820, Part II, p. 214,) thus writes, " It is possible that death may be the consequence of two generally co-existing causes; the one, chance, without previous disposition to death or deterioration; the other, a deterioration, or an increased inability to withstand destruction. If, for instance, there

be a number of diseases to which the young and old were equally liable, and likewise which should be equally destructive, whether the patient be young or old, it is evident that the deaths among the young and old, by such diseases, would be exactly in proportion of the number of young to the old; provided those numbers were sufficiently great for chance to have its play; and the intensity of mortality might then be said to be constant; and were there no other diseases but such as those, life of all ages would be of equal value, and the number of living and dying from a certain number living at a given earlier age, would decrease in geometrical progression, as the age increased by equal intervals of time; but if mankind be continually gaining seeds of indisposition, or in other words, an increased liability to death (which appears not to be an unlikely supposition with respect to a great part of life, though the contrary appears to take place at certain periods) it would follow that the number of living out of a given number of persons at a given age, at equal successive increments of age, would decrease in a greater ratio than the geometrical progression." In a following paragraph, Mr. Gompertz supposes the average exhaustions of a man's power to avoid death to be such, that at the end of equal infinitely small intervals of time, he would lose equal proportions of his remaining power to oppose destruction. This assumption is virtually to the effect, treated in the same manner as the other deductions of present work, that the sum of the integral rates of mortality shall always be the sum of a geometrical progression, or that the intensity of each period's mortality shall always be greater than that of preceding period by a certain proportion. If then we denote the annual rate of increase by r, the integral rate for the x^{th} year will be $\int mr^{x-1}$ and the sum of the integral rates $\int m \frac{r^x-1}{r-1}$, consequently, the proportion of survivors at the end of the x years will be represented by ${}^x l = \epsilon^{\int m \frac{r^x-1}{r-1}}$ which is of the form ${}^x l = \epsilon^{\int m \frac{r^x}{r-1}} \times \epsilon^{-\int m \frac{1}{r-1}}$, or $q^{r^x} \div q$ in which $q = \epsilon^{\int m \frac{1}{r-1}}$. It has been shown that whatever progression is assigned to the integral rates of mortality, the same progression prevails among the Napierian logarithms of the probabilities of living, and as all systems of logarithms are proportional, a similar progression must prevail among the common logarithms, consequently the common logarithms of the probabilities of living in connection with the preceding function will be in geometrical progression, and as the logarithm of the proportion surviving x years is the sum of the x logarithms of the intermediate probabilities of living, we obtain the equation $\lambda {}^x l = \lambda {}^1 l \frac{r^x-1}{r-1}$ as the fundamental source of other relations, the values of r and ${}^1 l$ being determinable from the data, x being variable at pleasure, and the integral rate of mortality equal to the common logarithm (considered as positive) of the probability of living, multiplied by the value of μ^{-1} or 2·3025851. Mr. Gompertz, according to his method of derivation, in showing the approximation of the results obtained from the exponential function, to those inserted in the prevailing Tables of Mortality, virtually determines the value of what we have denoted by r, from the ratio existing between the differences of the logarithms defining the first and last periods, from which the value of $\lambda {}^1 l$ can be so determined by the common properties of geometrical progression, that the sums of the graduated and original logarithms of the probabilities of living, and therefore of the proportions surviving, can be made to correspond. Mr. Galloway has adopted a similar method in his arrangement of the Experience of the Amicable Society. Mr. Edmonds, in his Life-Tables (1832), proposes that certain values of the ratio should

be considered as constants for all Tables of Mortality, and should be applied to a rate of mortality deducible from the particular data, not as a rate terminating, but as beginning, the first interval. As the assumption of a commencing or inceptive rate at the *origin* of the first interval gives rather a different form to the expression, it may be well in accordance with the preceding principles of present work to denote how such a rate may be deduced from any of the differential rates of mortality. The Napierian logarithm of the probability of surviving any period, has been shown, when considered with a positive sign, to indicate the integral mortality, and by aid of the Integral Calculus we observe, that the integral progresses in connection with the differential co-efficient. The sum, therefore, of the resulting increments will be the limit of the sum of the effects produced upon the inceptive rate. Hence putting $+\int m$ as the collective integral mortality for the whole number of periods, and denoting the commencing rate by c, that at the termination of the x periods will be cr^x.

$$\text{Consequently as } +\int m=\mu^{-1}\lambda^x l^{-1} \therefore \mu^{-1}\lambda^x l^{-1}=c\int r^x dx=c\frac{r^x-1}{\mu^{-1}\lambda r}$$

$$\therefore c=\mu^{-1}\lambda^x l\,\frac{\mu^{-1}\lambda r}{1-r^x} \therefore \lambda^x l=\frac{\mu^2 c}{\lambda r}(1-r^x) \therefore {}^x l=10^{\frac{\mu^2 c}{\lambda r}(1-r^x)}$$

The function of the fundamental form a^{b^c}, or any modification of it, is principally useful in cases where the Calculus of Finite Differences cannot be applied in determining and apportioning the results of the generic law of relation; but there seems no reason for assigning a specific law of relation, as for instance, that of geometrical progression, until it becomes almost a matter of necessity, either from paucity of data, or when a change in the previous generic law is to be anticipated, as in the *general* mortality of an assurance society, at ages greater than about 71, when the new admissions become too few to involve a continuation of the mixed generic law previously prevailing.

The functions a^{b^c} and cr^x in addition to their algebraical significance, may also have an interpretation given to them as results from algebraic geometry, or as equations to Transcendental Curves. This affinity of geometrical curves to the progressions displayed by rates of mortality, has induced more than one eminent writer to endeavour to generate a natural curve from the data presented; a method which though in principle approaching to philosophical precision, involves such uncertainties in the details, that even under the best treatment the results are with difficulty kept regular. Mr. Milne (Annuities 1815, Art. 180-705,) adopted this method in his arrangement of Dr. Heysham's Carlisle data, and accordingly traced one curve for the numbers living, and another for the numbers dying, by taking the enumeration-intervals of age as the abscissæ, and the relative numbers of the enumeration as ordinates, in the first instance to adjacent rectangular parallelograms, and then to similar areas bounded by a curve, having its origin from the preceding area; consequently the subdivision of the lines of abscissæ by ordinates into annual intervals afforded the comparative curvilinear areas representing the ratios of dying to living at all ages. Twelve years subsequently, (Parl. Rep. on Friendly Societies, 1827, p. 62,) Mr. Milne in allusion to the rates of mortality not always increasing with the age in the Carlisle Table so constructed, after partly explaining such deviations, as probably indicating some special law of nature, and partly as arising from accidental circumstances in the state of the population, the influence of which would have disappeared had the observations been on a much larger scale, with his usual candour, adds "and partly from

errors I may have committed in distributing the numbers given by the observations among the separate years of age, by a tedious tentative process founded on geometrical principles;" an admission, from so competent a calculator, that may fairly deter others from following a similar mode of construction. A method employed by writers who have attempted popular expositions of similar data, is to render each annual term, a "mean" of several others. To show that such a method should be confined to popular illustrations, it is only necessary to compare contiguous terms so computed. Thus

$$\frac{{}^{n+1}m+{}^{n}m+\ldots\ldots+{}^{2}m}{n} - \frac{{}^{n}m+{}^{n-1}m+\ldots\ldots+{}^{1}m}{n} = \frac{{}^{n+1}m-{}^{1}m}{n};$$ consequently the original difference between two terms is merely set aside, that an aliquot part of the difference between two others may be substituted.

In framing the fundamental Tables A and C, of the present work, the author has employed the Calculus of Finite Differences in determining the generic law from the Age of 21 to Age 71, and has continued such law for the remainder of Table A to produce the results of Table AA, the uniform principle of selection being supposed continuous to the extreme ages. The function, however, of a^{bc} as to survivors, or that of $\mu^{x-1}\lambda^{1}lr^{x-1}$ as to the integral rates of mortality has been employed in continuation of Table C, to produce the results of Table CC, because the *general* mortality in relation to the previous generic law cannot, after about the age of 71, be considered as any longer sustained by the mixture of new admissions.

Previously to illustrating the actual application of these principles to the assigned data, it may be useful to remark that the two methods virtually become one, if the orders of differences and the first term be taken, so as to approximate to a series, in geometrical progression; a sufficient number of orders being taken to render those neglected, close approximations to the limit zero. For the function in connection with the Calculus of Finite Differences, is of the form $(a+b)^n$ expanded by the binomial theorem; the coefficients being retained, and the commencing term and successive order of differences substituted for the terms involving the products of a and b, as

$$a^n b^0+{}^{n-1}ab^1+{}^{n-2}ab^2+{}^{n-3}ab^3+{}^{n-4}ab^4+\ldots\ldots\ldots\ldots\ldots\ldots+a^0b^n.$$

Now it is obvious that these products form a geometrical progression, the common ratio being $\frac{b}{a}$; and it is equally obvious that the successive powers of $(a+b)$ involving the coefficients are in geometrical progression; whence the alliance of the two functions, in connection with the conditions stated, is displayed, if the Calculus of Finite Differences be applied to the logarithms of the probabilities of living, and the function a^{bc} to the numbers surviving, or $\mu^{x-1}\lambda^{1}lr^{x-1}=cr^{x}$ to the integral rates of mortality. The above alliance, will be found on examination to receive a practical exemplification in the Table of Mortality inserted in the Appendix, pp. 167 to 169 of the Registrar General's Fifth Report, the origin of the similarity of results there presented being traceable to the reasons above explained.

In framing Tables of Mortality from assigned data by aid of the Calculus of Finite Differences, it is necessary that the terms employed to determine the presumed generic law should be *collective* terms, in order that they may really involve some law of relation, for without this proviso, it is evident no law of connection could reasonably be supposed to exist between them, except as differences arising from some preceding series. The proportions surviving from the commencing age to subsequent periods of life, are

obviously collective terms and display a primary scale of relation; as do also the relative logarithms, which have been already shown to indicate the progression of the collective integral mortalities. In employing these, however, it is necessary that the values adopted as terminations of the particular intervals, should afford by interpolation, annual rates of mortality increasing with the age, to adapt the Table for practical purposes.

The usual method of effecting this, has been to take actual terms at short intervals, as for instance of four or five years from the original data, and then to adjust these, by such corrections as will ensure the desired regularity. This has been the method adopted both by Mr. Woolhouse, in his Table of Indian Mortality, (1839,) and in that of the Adjusted Table (G_r) presented by the Committee of Actuaries, already referred to in the Introduction, as embodying the results of the Combined Town and Country Experience. Under such excellent superintendence, the results in botb instances are admirably arranged, but as a general principle, it appears desirable, that any method involving personal adjustments should be avoided, not only because these adjustments are left variable, according to the opinions of different calculators, even in connection with the same data, but also, because the primary attempt at precision, indicated by the choice of small intervals, is eventually neutralized, if such values, though selected from the original data, have to be again adjusted and thus cease to represent actual elements of coincidence with the primary values.

A preferable method, when available, appears to result, by taking the intervals sufficiently large in the first instance, to allow the values thus taken from the original data, to remain persistent in the graduated Table, and thus secure certain points of coincidence between the calculated and original values. The intermediate values may thus be wholly determined by the function of interpolation adopted; and so enable the actual principle of construction to be distinctly stated, and the results capable of verification by any other calculator from the same data. However irregular such data may be, certain periods can generally be found on trial, which will allow of increasing rates of mortality being interpolated by the Calculus of Finite Differences, or by the function a^{b^c} already explained, and as decennial periods are convenient elements in data generally, if quinquennial periods do not supply the necessary regularity of increase, the decennial intervals appear preferable to any intermediate subdivision.

The application, of the methods just indicated, to the purposes of present work, will be found elucidated in the succeeding Section.

SECTION II.

ON THE APPLICATION OF THE PRECEDING METHODS TO THE ASSIGNED DATA OF PRESENT WORK.

After the explanations of the preceding Section, the following values derived from the assigned data inserted in p. 9, and relating to Table A, will be readily understood. The first column shows the age; the second the logarithms of the proportions surviving to such age; the third the arithmetical complements, which if multiplied by μ^{-1} or 2·3025851 would represent, as has been shown, the collective integral rates of mortality from the first age; such multiplication, however, is unnecessary, as the character of the progression would not be thereby altered. The fourth, fifth, sixth, seventh and eighth columns display the first, second, third, fourth and fifth order of differences.

VALUES DERIVED FROM THE ASSIGNED DATA RELATING TO TABLE A. *Vide p. 9.*

Age	* $\lambda^x l$	$\kappa^x l$	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5
21	·0000000+0·	0·0000000	0·0271035	0·0055333	0·0117196	0·0149128	0·0048856
31	·9728965−1·	0·0271035	0·0326368	0·0172529	0·0266324	0·0197984	
41	·9402597−1·	0·0597403	0·0498897	0·0438853	0·0464308		
51	·8903700−1·	0·1096300	0·0937750	0·0903161			
61	·7965950−1·	0·2034050	0·1840911				
71	·6125039−1·	0·3874961					

The function of interpolation afforded by the Calculus of Finite Differences, will be as follows;

$$\kappa^x l = 0 + n\Delta^1 + n.\frac{n-1}{2}\Delta^2 + n.\frac{n-1}{2}.\frac{n-2}{3}\Delta^3 + n.\frac{n-1}{2}.\frac{n-2}{3}.\frac{n-3}{4}\Delta^4 + n.\frac{n-1}{2}.\frac{n-2}{3}.\frac{n-3}{4}.\frac{n-4}{5}.\Delta^5$$

In which $\kappa^x l$ is supposed to represent the nth term; 0 the first term; and n to equal $\frac{x}{10}$.

It has been found preferable, on the present occasion, in applying the preceding expression, to calculate the values afforded by the function for each age separately, and thus avoid neglected decimals, rather than to attempt approximations, by assuming orders of differences for annual intervals, until indeed the close of the series renders such a process necessary for continuation. The following example for the Age of 22, or for the first interval, affords a type of the mode adopted for the other ages, and is not only convenient for the particular purpose, but displays the values in relation to each succeeding order of differences.

* The mantissæ are made in the above Table and elsewhere in present work, to precede the negative characteristics of the logarithms for the convenience of printing; a notation it will be seen results, perhaps preferable to that usually adopted, the two decimal points acting as parentheses and readily distinguishing the logarithms from ordinary numbers; a point of some importance in mixed calculations, involving an equal number of figures both in the mantissæ of logarithms and decimal parts of other numbers. In the summation &c. of logarithms so expressed, the characteristics as usual should obviously be treated last.

Age.	n $\frac{n-1}{2}$ &c.	λ λ &c.	$\Sigma\lambda$ $\Sigma\lambda$ &c.	$\Sigma\lambda+\lambda\Delta^1$ $\Sigma\lambda+\lambda\Delta^2$ &c.	N N &c.	ΣN ΣN &c.
22	+0·1000000	·0000000-1·	·0000000-1·	·4330254-3·	+0·0027103 50	0·0027103 50
	—0·4500000	·6532125-1·	·6532125-2·	·3961967-4·	—0·0002489 985	0·0024613 515
	—0·6333333	·8016324-1·	·4548449-2·	·5237578-4·	+0·0003340 087	0·0027953 602
	—0·7250000	·8603380-1·	·3151829-2·	·4887421-4·	—0·0003081 357	0·0024872 245
	—0·7800000	·8920946-1·	·2072775-2·	·8961954-5·	+0·0000787 4000	0·0025659 645

The value at end of last column, or 0·0025660, is obviously that of $\kappa^1 l$ $\therefore \lambda^1 l =$ ·9974340-1· as will be found inserted at Age 22, in the Graduated Column of the Table in p. 9. The values appropriated to the other ages, extending to Age 71 inclusive, may be traced in a similar manner.*

The last interval in each decennial being a remainder-interval, is scarcely a result of the general relation, consequently the continuation of the series beyond the Age of 71, will be best fulfilled by deducing five orders of *annual* differences, from the six intermediate values obtained in relation to the Ages of 63 to 68. The following Table is constructed by such means.

Age.	$\lambda^x l$	$\kappa^x l$	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5
63	·7688913-1·	0·2311087	0·0153255	0·0010669	0·0000663	0·0000029	0·0000000
64	·7535658-1·	0·2464342	0·0163924	0·0011332	0·0000692	0·0000029	
65	·7371734-1·	0·2628266	0·0175256	0·0012024	0·0000721		
66	·7196478-1·	0·2803522	0·0187280	0·0012745			
67	·7009198-1·	0·2990802	0·0200025				
68	·6809173-1·	0·3190827					

It thus appears that for annual intervals the fourth order of differences may be considered constant and equal to 0·0000029, and consequently the fifth order be represented by zero.

Recommencing then with Age 68, the progression to Age 73, will be as denoted in following Table, the values at more advanced ages being determined by continuing the process of considering the fourth order of differences constant and equal to 0·0000029.

Age.	$\lambda^x l$	$\kappa^x l$	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5
68	·6809173-1·	0·3190827	0·0213521	0·0014266	0·0000800	0·0000029	0·0000000
69	·6595661-1·	0·3404339	0·0227778	0·0015066	0·0000829	0·0000029	
70	·6367883-1·	0·3632117	0·0242844	0·0015895	0·0000858		
71	·6125039-1·	0·3874961	0·0258739	0·0016753			
72	·5866300-1·	0·4133700	0·0275492				
73	·5590808-1·	0·4409192					

The complete Table of data and calculations, upon which Table A is founded, is accordingly presented in following page.

* It facilitates such calculations to remember, that as the values of the coefficient n, proceed in arithmetical progression by steps of 0·1 for each additional age, so must those of $\frac{n-1}{2} = \left(\frac{n}{2} - \frac{1}{2}\right)$ by steps of 0·05; those of $\frac{n-2}{3} = \left(\frac{n}{3} - \frac{2}{3}\right)$ by 0·0333333; those of $\frac{n-3}{4}$ by 0·025; and those of $\frac{n\,4}{5}$ by 0·02. The various orders of coefficients are thus readily ascertained, and by the addition of the logarithms, the relative products easily computed, as shown in above example.

TABLE OF DATA AND CALCULATIONS UPON WHICH TABLE A, p.vi. IS FOUNDED. *Vide p.* 8.

Complete Age.	DATA. Four times the Number exposed to Risk of Mortality.	DATA. Four times the number of Deaths	IRREGULAR RATIOS. Mortality ℘ Cent.	IRREGULAR RATIOS. Mortality ℘ 1000, at Ten Ages.	IRREGULAR RATIOS. $_1l$	IRREGULAR RATIOS. $\lambda_1 l$	IRREGULAR RATIOS. λl	Complete Age.	GRADUATED RATIOS. λl	GRADUATED RATIOS. $\lambda_1 l$	GRADUATED RATIOS. $_1l$	TABLE A. Mortality ℘ Cent.	TABLE A. Mortality ℘ 1000, at Ten Ages.
21	2394	20	0·8354		·991646	·9963566-1·	*·0000000+0·	21	*·0000000+0·	·9974340-1·	·994109	0·5891	
22	3232	18	0·5569		·994431	·9975746	·9963566—1·	22	·9974340—1·	·9974017	·994035	0·5965	
23	3760	36	0·9574		·990426	·9958220	·9939312	23	·9948357	·9973707	·993964	0·6036	
24	4121	12	0·2912		·997088	·9987335	·9897532	24	·9922064	·9973402	·993894	0·6106	
25	5546	42	0·7573		·992427	·9966986	·9884867	25	·9895466	·9973098	·993825	0·6175	
26	5449	32	0·5873		·994127	·9974419	·9851853	26	·9868564	·9972786	·993753	0·6247	
27	6496	62	0·9544		·990456	·9958352	·9826272	27	·9841350	·9972461	·993679	0·6321	
28	6661	38	0·5705		·994295	·9975153	·9784624	28	·9813811	·9972112	·993599	0·6401	
29	7485	38	0·5077		·994923	·9977895	·9759777	29	·9785923	·9971731	·993512	0·6488	
30	7987	16	0·2003	6·2184	·997997	·9991293	·9737672	30	·9757654	·9971311	·993416	0·6584	6·2214
31	8010	50	0·6242		·993758	·9972806	*·9728065	31	*·9728965	·9970840	·993308	0·6692	
32	8001	30	0·3750		·996250	·9983683	·9701771	32	·9699805	·9970310	·993187	0·6813	
33	8105	60	0·7403		·992597	·9967730	·9685454	33	·9670115	·9969708	·993049	0·6951	
34	8324	56	0·6728		·993272	·9970682	·9653184	34	·9639823	·9969026	·992893	0·7107	
35	8380	54	0·6444		·993556	·9971923	·9623866	35	·9608849	·9968251	·992716	0·7284	
36	8164	76	0·9309		·990691	·9959382	·9595789	36	·9577100	·9967370	·992515	0·7485	
37	8127	50	0·6152		·993848	·9973200	·9555171	37	·9544470	·9966371	·992287	0·7713	
38	7495	76	1·0141		·989859	·9955734	·9528371	38	·9510841	·9965244	·992029	0·7971	
39	7363	58	0·7877		·992123	·9965655	·9484105	39	·9476085	·9963971	·991738	0·8262	
40	7777	84	1·0801	7·4847	·989199	·9952837	·9449760	40	·9440056	·9962541	·991412	0·8588	7·4866
41	6840	36	0·5263		·994737	·9977083	*·9402597	41	*·9402597	·9960939	·991046	0·8954	
42	6585	44	0·6682		·993318	·9970883	·9379680	42	·9368536	·9959149	·990638	0·9362	
43	6483	112	1·7276		·982724	·9924316	·9350563	43	·9322685	·9957158	·990184	0·9816	
44	6276	62	0·9879		·990121	·9956882	·9274879	44	·9279843	·9954947	·989680	1·0320	
45	6059	82	1·3534		·986466	·9940821	·9231761	45	·9234790	·9952502	·989123	1·0877	
46	5184	48	0·9259		·990741	·9959601	·9172582	46	·9187292	·9949806	·988509	1·1491	
47	5245	56	1·0677		·989323	·9953381	·9132183	47	·9137098	·9946839	·987834	1·2166	
48	4853	86	1·7721		·982279	·9922349	·9085564	48	·9083937	·9943587	·987094	1·2906	
49	4913	48	0·9770		·990230	·9957361	·9007913	49	·9027524	·9940029	·986286	1·3714	
50	4404	62	1·4078	11·4139	·985922	·9938426	·8965274	50	·8967553	·9936147	·985405	1·4595	11·4201
51	3968	36	0·9073		·990927	·9960417	*·8903700	51	*·8903700	·9931922	·984447	1·5553	
52	3889	60	1·5428		·984572	·9932475	·8864117	52	·8835622	·9927334	·983407	1·6593	
53	3512	50	1·4237		·985763	·9937725	·8796592	53	·8762956	·9922362	·982282	1·7718	
54	3037	66	2·1732		·978268	·9904578	·8734317	54	·8685318	·9916985	·981067	1·8933	
55	3014	62	2·0571		·979429	·9909730	·8638895	55	·8602303	·9911185	·979757	2·0243	
56	2769	54	1·9502		·980498	·9914467	·8548625	56	·8513488	·9904936	·978349	2·1651	
57	2434	46	1·8899		·981101	·9917137	·8463092	57	·8418424	·9898218	·976836	2·3164	
58	2206	58	2·6292		·973708	·9884288	·8380229	58	·8316642	·9891008	·975216	2·4784	
59	2150	44	2·0465		·979535	·9910199	·8264517	59	·8207650	·9883282	·973483	2·6517	
60	1875	88	4·6933	21·3132	·953067	·9791284	·8174716	60	·8090932	·9875018	·971632	2·8368	21·3524
61	1523	62	4·0709		·959291	·9819504	*·7965950	61	*·7965950	·9866189	·969659	3·0341	
62	1394	26	1·8651		·981349	·9918235	·7785454	62	·7832139	·9856774	·967559	3·2441	
63	1338	88	2·8401		·971599	·9874871	·7703689	63	·7688013	·9846745	·965327	3·4673	
64	1106	52	4·7016		·952984	·9790856	·7578560	64	·7535658	·9836076	·962959	3·7041	
65	1012	18	1·7787		·982213	·9922057	·7369416	65	·7371734	·9824744	·960449	3·9551	
66	738	22	2·9810		·970190	·9868568	·7291473	66	·7106478	·9812720	·957794	4·2206	
67	506	12	2·3715		·976285	·9895766	·7160041	67	·7009198	·9799975	·954987	4·5013	
68	316	24	7·5949		·924051	·9656960	·7055807	68	·6809173	·9786488	·952026	4·7974	
69	176	6	3·4091		·965909	·9849303	·6712767	69	·6595661	·9772222	·948904	5·1096	
70	188	18	9·5745	41·1874	·904255	·9562909	·6562130	70	·6367883	·9757156	·945618	5·4382	41·4718
71	...	..	.....	.....	.....	.....	*·6125039	71	*·6125039	.....	.....	.....	.....

CONTINUATION OF THE GRADUATED RATIOS, λl and $\lambda_1 l$ vide p. 8.

Age.	λl	$\lambda_1 l$	Age.	λl	$\lambda_1 l$	Age.	λl	$\lambda_1 l$
71	·6125039-1·	·9741261-1·	81	·2671234-1·	·9530336-1·	91	·6015504-2·	·9204611-1·
72	·5866300	·9724508	82	·2201570	·9503408	92	·5820115	·9164608
73	·5590808	·9706868	83	·1704978	·9475303	93	·4984723	·9123138
74	·5297676	·9688312	84	·1180281	·9445992	94	·4107861	·9080172
75	·4985988	·9668811	85	·0626273	·9415446	95	·3188033	·9035681
76	·4654799	·9648336	86	·0041719-2·	·9383636	96	·2223714	·8989636
77	·4303135	·9626858	87	·9425355	·9350533	97	·1213350	·8942008
78	·3929993	·9604348	88	·8775888	·9316108	98	·0155358	·8892768
79	·3534341	·9580777	89	·8091996	·9280332	99	·9048126-3·	·8841887
80	·3115118	·9556116	90	·7372328	·9243176	100	·7890013	.....

The fundamental construction of Table A, or that relating to the first year's mortality after selection, being thus explained, it now remains to show the application of the same method to the general mortality represented by Table C; the assigned data being inserted in p. 11.

VALUES DERIVED FROM THE ASSIGNED DATA RELATING TO TABLE C. *Vide p.* 11.

Age.	$\lambda^x l$	$\kappa^x l$	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5
21	·0000000+0·	0·0000000	0·0354713	0·0070016	0·0091079	0·0192116	0·0129508
31	·9645287−1·	0·0354713	0·0424729	0·0161095	0·0283195	0·0321624	
41	·9220558−1·	0·0779442	0·0585824	0·0444290	0·0604819		
51	·8634734−1·	0·1365266	0·1030114	0·1049109			
61	·7604620−1·	0·2395380	0·2079223				
71	·5525397−1·	0·4474603					

The following is an example of the determination of the value for age 22, in same manner as that already given in relation to Table A, in the preceding page.

Age.	n	λ	$\Sigma\lambda$	$\Sigma\lambda+\lambda\Delta^1$	N	ΣN
	$\frac{n-1}{2}$	λ	$\Sigma\lambda$	$\Sigma\lambda+\lambda\Delta^2$	N	ΣN
	&c.	&c.	&c.	&c.	&c.	&c.
22	+0·1000000	0000000-1·	·0000000-1·	·5498771-3·	+0·0035471 30	0·0035471 30
	−0·4500000	·6532125-1·	·6532125-2·	·4984098-4·	−0·0003150 719	0·0032320 581
	−0·6333333	·8016324-1·	·4548449-2·	·4142632-4·	+0·0002595 752	0·0034916 333
	−0·7250000	·8603380-1·	·3151829-2·	·5987465-4·	−0·0003969 598	0·0030946 735
	−0·7800000	·8920946-1·	·2072775-2·	·3195741-4·	+0·0002087 248	0·0033033 983

The value at end of last column or 0·0033034 is obviously that of $\kappa^1 l$ ∴ $\lambda^1 l$ = ·9966966-1· as will be found inserted at Age 22 in the Graduated Column of the Table opposite.

It has been already remarked in p. 5, that the admissions after the Age of 71, are too few to admit of the supposition that the same mixed generic law will continue to prevail in the general mortality after that age as prevailed previously. The present, therefore, is one of those cases in which the fundamental function a^{b^c} already explained, is especially valuable; for though admissions virtually end at about the Age of 71, yet a more or less sufficient number of lives, previously assured, are exhibited by the data as passing from the Ages of 71 &c. to Age 81, to indicate the progressional mortality to be expected among such lives in general. Having therefore determined the mixed generic law, previously prevailing, by aid of the Calculus of Finite Differences, and ascertained as one of its results, the Napierian logarithm of the probability of living from the Age of 70 to 71, or the integral mortality during that period, we can readily estimate the value of r in the equation $\mu^{-1}\lambda^x l = fm\frac{r^x-1}{r-1}$ by taking Age 70 as the commencing age, and determining the collective integral rate of mortality that actually prevailed from the Age of 71 to 81, as shown by the assigned data. But it has already been indicated that the value of r will remain the same whether Napierian or common logarithms be employed; consequently a more convenient form of the above equation will be that of p. 3, or $\lambda^x l = \lambda^1 l\frac{r^x-1}{r-1}$.

From the Table in p. 11, it appears that the value of $\lambda^x l$ at the Age of 81, compared with that at the Age of 71, is ·0903770-1·—·5525397-1· = ·5378373-1· whose arithmetical complement is 0·4621627, and that the value of λ, l, or the logarithm of the probability

TABLE OF DATA AND CALCULATIONS UPON WHICH TABLE C, p. xiii. IS FOUNDED, p. 10.

Complete Age.	DATA. Four times the Number exposed to Risk of Mortality.	DATA. Four times the number of Deaths.	IRREGULAR RATIOS. Mortality ℘ Cent.	IRREGULAR RATIOS. Mortality ℘ 1000, at Ten Ages.	IRREGULAR RATIOS. $_1l$	IRREGULAR RATIOS. $\lambda_1 l$	IRREGULAR RATIOS. λl	Complete Age.	GRADUATED RATIOS. λl	GRADUATED RATIOS. $\lambda_1 l$	GRADUATED RATIOS. $_1l$	TABLE C. Mortality ℘ Cent.	TABLE C. Mortality ℘ 1000, at Ten Ages.
21	9152	106	1·1582		·988418	·9949406-1·	*·0000000+0·	21	*·0000000+0·	·9966966-1·	·992423	0·7577	
22	11116	72	0·6475		·993525	·9971788	·9949406—1·	22	·9966966—1·	·9966469	·992309	0·7691	
23	14199	134	0·9437		·990563	·9958821	·9921194	23	·9933435	·9965950	·992190	0·7810	
24	18038	138	0·7649		·992351	·9966553	·9880015	24	·9899385	·9965416	·992068	0·7932	
25	22432	172	0·7668		·992332	·9966570	·9846668	25	·9864801	·9964865	·991943	0·8057	
26	27204	218	0·8015		·991985	·9965051	·9813238	26	·9829666	·9964304	·991814	0·8186	
27	32176	294	0·9135		·990865	·9960145	·9778289	27	·9793970	·9963728	·991683	0·8317	
28	37585	322	0·8566		·991434	·9962639	·9738434	28	·9757698	·9963141	·991549	0·8451	
29	43692	232	0·5310		·994690	·9976878	·9701073	29	·9720839	·9962536	·991411	0·8589	
30	49910	374	0·7493	8·1330	·992507	·9967336	·9677951	30	·9683375	·9961912	·991268	0·8732	8·1342
31	55565	410	0·7378		·992622	·9967839	*·9645287	31	*·9645287	·9961264	·991121	0·8879	
32	61109	430	0·7037		·992963	·9969331	·9613126	32	·9606551	·9960584	·990965	0·9035	
33	65862	606	0·9201		·990799	·9959856	·9582457	33	·9567135	·9959866	·990801	0·9199	
34	70195	636	0·9059		·990941	·9960478	·9542313	34	·9527001	·9959102	·990627	0·9373	
35	74120	850	1·1469		·988531	·9949903	·9502791	35	·9486103	·9958279	·990439	0·9561	
36	77739	790	1·0161		·989839	·9955646	·9452694	36	·9444382	·9957388	·990236	0·9764	
37	80627	844	1·0468		·989532	·9954299	·9408340	37	·9401770	·9956415	·990014	0·9986	
38	82227	816	0·9924		·990076	·9956685	·9362639	38	·9358185	·9955346	·989771	1·0229	
39	84272	868	1·0300		·989700	·9955036	·9319324	39	·9313531	·9954167	·989502	1·0498	
40	85768	1056	1·2312	9·7309	·987688	·9946198	·9274360	40	·9267698	·9952860	·989204	1·0796	9·7320
41	86046	902	1·0482		·989518	·9954237	*·9220558	41	*·9220558	·9951407	·988873	1·1127	
42	86271	994	1·1522		·988478	·9949670	·9174795	42	·9171965	·9949791	·988506	1·1494	
43	86525	1000	1·1557		·988443	·9949516	·9124465	43	·9121756	·9947988	·988095	1·1905	
44	86204	1010	1·1717		·988283	·9948813	·9073981	44	·9069744	·9945979	·987638	1·2362	
45	85522	1128	1·3189		·986811	·9942389	·9022794	45	·9015723	·9943739	·987129	1·2871	
46	83772	978	1·1674		·988326	·9949002	·8965133	46	·8959462	·9941246	·986563	1·3437	
47	82272	1110	1·3493		·986507	·9941002	·8914135	47	·8900708	·9938471	·985932	1·4068	
48	80192	1398	1·7433		·982567	·9928622	·8855187	48	·8839179	·9935390	·985233	1·4767	
49	78247	1226	1·5668		·984332	·9931416	·8778759	49	·8774569	·9931973	·984458	1·5542	
50	76300	1314	1·7221	13·3956	·982779	·9924559	·8710175	50	·8706542	·9928192	·983602	1·6398	13·3971
51	73192	1268	1·7325		·982675	·9924099	*·8634734	51	*·8634734	·9924014	·982656	1·7344	
52	70466	1376	1·9525		·980475	·9914365	·8558833	52	·8558748	·9919408	·981614	1·8386	
53	67512	1276	1·8901		·981099	·9917128	·8473198	53	·8478156	·9914342	·980470	1·9530	
54	64411	1224	1·9003		·980997	·9916677	·8390326	54	·8392498	·9908778	·979215	2·0785	
55	61287	1466	2·3919		·976081	·9894859	·8307003	55	·8301276	·9902683	·977841	2·2159	
56	58261	1422	2·4407		·975593	·9892687	·8201862	56	·8203959	·9896017	·976341	2·3659	
57	55097	1262	2·2903		·977097	·9899377	·8094549	57	·8099976	·9888744	·974708	2·5292	
58	52322	1394	2·6643		·973357	·9882722	·7993926	58	·7988720	·9880822	·972931	2·7069	
59	49658	1504	3·0285		·969715	·9866442	·7876648	59	·7869542	·9872210	·971004	2·8996	
60	46464	1458	3·1381	23·4292	·968619	·9861530	·7743090	60	·7741752	·9862868	·969818	3·1082	23·4302
61	43012	1416	3·2922		·967078	·9854615	*·7604620	61	*·7604620	·9852749	·966663	3·3337	
62	39686	1310	3·3006		·966994	·9854238	·7459235	62	·7457369	·9841809	·964231	3·5769	
63	36618	1252	3·4189		·965811	·9848922	·7313473	63	·7299178	·9830001	·961612	3·8388	
64	33729	1530	4·5360		·954640	·9798396	·7162395	64	·7129179	·9817280	·958800	4·1200	
65	30086	1352	4·4953		·955947	·9804338	·6960791	65	·6946459	·9805595	·955784	4·4216	
66	27666	1350	4·8789		·951211	·9782769	·6765129	66	·6750054	·9788892	·952553	4·7447	
67	24349	1062	4·3614		·956386	·9806332	·6547898	67	·6538946	·9773127	·949101	5·0899	
68	21538	1190	5·5246		·944754	·9753187	·6354230	68	·6312073	·9756243	·945419	5·4581	
69	18915	1082	5·7188		·942812	·9744251	·6107417	69	·6068316	·9738183	·941496	5·8504	
70	16466	1192	7·2374	46·6741	·927626	·9673729	·5851668	70	·5806499	·9718898	·937324	6·2676	46·7017
71	.....	....	.....	.....	.....	.....	*·5525397	71	*·5525397	.....	.....	.....	.....

CONTINUATION OF THE PRECEDING TABLE, vide p. 10.

71	14044	1012	7·2079		·927921	·9675110-1·	*·5525397-1·	71	*·5525397-1·	·9693956-1·	·931956	6·8044	
72	12029	834	6·9326		·930674	·9687976	·5200507	72	·5219353	·9666800	·926147	7·3853	
73	10334	814	7·8799		·921201	·9643544	·4888483	73	·4886153	·9637235	·919864	8·0136	
74	8772	782	8·9146		·910854	·9594488	·4532027	74	·4523388	·9605047	·913071	8·6929	
75	7314	688	9·4066		·905934	·9570965	·4126515	75	·4128435	·9570003	·905733	9·4267	
76	6058	662	10·9277		·890723	·9497427	·3697480	76	·3698438	·9531849	·897811	10·2189	
77	4927	504	10·2294		·897706	·9531341	·3194907	77	·3230287	·9490310	·889264	11·0736	
78	4052	510	12·5864		·874136	·9415790	·2726248	78	·2720597	·9445085	·880052	11·9948	
79	3170	414	13·0600		·869400	·9392196	·2142038	79	·2165682	·9395847	·890131	12·9869	
80	2531	342	13·5124	100·6575	·8648755	·9369536	·1534234	80	·1561529	·9342441	·859456	14·0544	100·6515
81	....	...	.....	.....	.....	.....	*·0903770	81	*·0903770	.....	.....	.....	.....

of living one year at Age 70, as determined by the Calculus of Finite Differences is ·9718898-1· whose complement is 0·0281102; consequently the value of $\kappa^{\omega}l$ from the Age of 70 to 81 in the graduated scale, will have to be the sum of these complements, or 0·4902729; by substitution, therefore, the equation becomes $0{\cdot}4902729 = 0{\cdot}0281102\ \frac{r^{11}-1}{r-1}$ from which, by the Theory of Equations of n Dimensions, the corresponding value of λr can be found, or $\lambda r = {\cdot}0369203 + 0{\cdot}$. The value of λr being thus determined, the means of continuing the series is obviously that of simple geometrical progression, applied to the logarithms of the probabilities of living, which when Napierian Logarithms are used or the common logarithms multiplied by $\mu^{-1} = 2{\cdot}3025851$, have been shown, with a change of sign, to represent the integral rates of mortality, the ratio of progression being the same whether Napierian or common logarithms be used; consequently the factor μ^{-1} may be neglected as a constant, or only used when the integral mortality itself is required in value. The logarithm of the probability of living one year at Age 70 being ·9718898-1· that at Age 71 will therefore be $(\cdot 9718898\text{-}1\cdot)r$; that at Age 72, $(9718898\text{-}1\cdot)r^2$; from which the construction of the remainder of the Table is obvious, the method of calculation being, to successively add to $\lambda(\kappa_1 l)$ the logarithm of r from age to age, by which process $\lambda_1 l$ and $\lambda^{\omega} l$ are determined for each age. Thus at Age 70; $\kappa_1 l = 0{\cdot}0281102 \therefore \lambda(\kappa_1 l) =$ ·4488639-2· to which, if $\lambda r = {\cdot}0369203 + 0{\cdot}$ be added, the sum or ·4857842-2· is the value of $\lambda\ (\kappa_1 l)$ at Age 71; consequently $\kappa_1 l = 0{\cdot}0306044$ or $\lambda_1 l = 0{\cdot}9693956\text{-}1{\cdot}$ as inserted in preceding Table. The continuation, therefore, of the graduated ratios will be as follows.

CONTINUATION OF THE GRADUATED RATIOS λl AND $\lambda_1 l$.

Age.	λl	$\lambda_1 l$	Age.	λl	$\lambda_1 l$	Age.	λl	$\lambda_1 l$	Age.	λl	$\lambda_1 l$
81	·0903770-1·	·9283878-1·	86	·6628815-2·	·8904560-1·	91	·0089488-2·	·8324323-1·	96	·0086389-3·	·7436746-1·
82	·0187648	·9220336	87	·5533375	·8807361	92	·8413811-3·	·8175640	97	·7523135-4·	·7209307
83	·9407984-2·	·9151156	88	·4340736	·8701538	93	·6589451	·8013765	98	·4732442	·6961689
84	·8559140	·9075838	89	·3042274	·8586325	94	·4603216	·7837525	99	·1694131	·6692098
85	·7634978	·8993837	90	·1628599	·8460889	95	·2440741	·7645648	100	·8386229-5·	

The values of the Expectation of Life at Ages after 70 may also be inserted here, to complete the Tables AA and CC, of p. xiv.

CONTINUATION OF THE "EXPECTATION OF LIFE" IN RELATION TO TABLE AA, p. xiv.

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.	Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
71	10·23	76	8·35	81	6·77	86	5·44	91	4·22	96	2·76
72	9·83	77	8·01	82	6·49	87	5·19	92	3·97	97	2·36
73	9·44	78	7·69	83	6·22	86	4·95	93	3·71	98	1·87
74	9·06	79	7·37	84	5·95	89	4·71	94	3·42	99	1.27
75	8·70	80	7·07	85	5·69	90	4·47	95	3·11	100	0·50

CONTINUATION OF THE "EXPECTATION OF LIFE" IN RELATION TO TABLE CC, p. xiv.

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.	Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
71	8·17	76	6.14	81	4·51	86	3·24	91	2·29	96	1·56
72	7·74	77	5·78	82	4·23	87	3·03	92	2·13	97	1·41
73	7·31	78	5·44	83	3·96	88	2·82	93	2·98	98	1·23
74	6·90	79	5·12	84	3·71	89	2·63	94	1·84	99	0·97
75	6·52	80	5·80	85	3·47	90	2·46	95	1·70	100	0·50

SECTION III.

ON THE MEANS OF VERIFYING THE ASSIGNED DATA OF PRESENT WORK, BY COMPARISON WITH THE RELATIVE "EXPERIENCE."

During the collection and arrangement of the necessary materials for a former publication, "On the Rise and Progress of the Doctrine of Life-Contingencies in England," (1844), the author became more and more impressed with the view, that Life-Assurance had gradually been made to depend nearly wholly upon mathematical demonstrations, and consequently, that Tables of Mortality solely depending for their adoption upon the authority of the authors framing them, would eventually be discarded; not so much from want of sufficient care in their construction, as from want of the subsidiary means of testing their adherence to the data upon which such Tables might be said to be founded. In accordance with this view, all modern publications on these subjects must principally depend for even ordinary acceptance, upon the attention paid to such details. Influenced by these considerations, having explained in the preceding Sections the principles of calculation adopted, and their adaptation to the assigned data, the author proceeds to offer as the subject of the concluding Section, such examples of the arrangements that have been made of the original materials, for the purposes of the present work, as will enable any calculator interested in the subject to verify the assigned data by comparison with the relative "Experience."*

* The Experience of the Equitable was printed in 1834 under the superintendance of the Society's Actuary, Mr. Arthur Morgan, and extends, as to mortality, to the close of the year 1829;—that of the Amicable in 1841, under the superintendence of the Society's Registrar, Mr. Galloway, extends to April 5th of the same year.—Either of these "Experiences" are easily obtained upon proper application to the gentlemen just mentioned.

The "Experience" of the fifteen Combined Offices was collected and printed at the expense of the following Life Assurance Societies, collectively, and is therefore also easily available. The Achilles, Albion, Alfred, *Alliance, Argus, Asylum, Atlas, Australasian, Britannia, *British Commercial, Church of England, Clerical Medical, *Crown, Dissenters', *Economic, Edinburgh, English and Scottish Law, European, Family Endowment, Farmers', Globe, *Guardian, Hand in Hand, Hope, *Imperial, *Law Life, Licensed Victuallers', London Assurance Corporation, *London Life, Medical Invalid, Metropolitan, Minerva, National, National Loan Fund, National Mercantile, North of Scotland, *Norwich Union, Palladium, Pelican, *Promoter, Protector, Reliance, Royal Exchange, Royal Naval and Military, Scottish Equitable, Scottish Provident, Scottish Union, *Scottish Widow's Fund, Standard, *Sun, Union, *Universal, *University, Victoria, Western, Westminster and General, West of England, York and London. The fifteen Contributing Societies are denoted by an asterisk; the following gentlemen, chiefly connected with them, forming the Committee of Actuaries under whose superintendence the Returns were arranged and printed:—Messrs. Ansell, Davies, Downes, Gompertz, Kirkpatrick, Milne, Rainbow, and Woolhouse; Messrs. Christie and Ingall acting as Secretaries. The resulting Experience was printed in 1843 and extends to the close of the year 1837.

Although the time and attention necessary for producing the present work at once proclaim the author's appreciation of the utility of the data thus made public, he cannot resist the opportunity of adding his testimony to that of others as to the accuracy and skilful arrangement of the details, and of expressing his belief that it is the liberality on the part of Assurance Societies, and the ability of those connected with them, of which such publications of data are only a type, that have caused the principles of Life Assurance to take deeper root in our own country than elsewhere, although in other countries such principles are practically, perhaps of even more political and social importance.

EXAMPLE,—IN RELATION TO TABLE A, SHOWING THE METHOD OF VERIFYING THE ASSIGNED DATA BY REFERENCE TO THE RELATIVE "EXPERIENCE."

Classes.	I. Age.	II. Assurances comprised.	III. Assurances Discontinued.	IV. Assurances Current.	V. Assurances becoming Claims.	VI. Total not passing on.	VII. 2(II-VI).	VIII. V+VI+VII +Eq: IV.	IX. 2(V).	X. Mortality ℘ Cent.
A(1)+A(4)	21 (current)	179	6	9	1	16	,,	,,	,,	,,
A(2)+A(5)	,, ,,	107	1	9	1	11	,,	,,	,,	,,
A(3)+A(6)	,, ,,	36	3	2	1	6	,,	,,	,,	,,
A(1)+A(4)	21 (complete)	163	78	6	4	88	,,	,,	,,	,,
A(2)+A(5)	,, ,,	96	34	6	0	40	,,	,,	,,	,,
A(3)+A(6)	,, ,,	30	14	3	0	17	,,	,,	,,	,,
Equitable	,, ,,	373	4	0	1	5	,,	,,	,,	,,
Totals.....		662	130	15	6	150	1024	1179	10	,,
Totals (doubled).....		1324	260	30	10	300	2048	2358	20	,,
Amicable (doubled) ..		19	1	1	0	2	34	36	0	,,
Final Totals		1343	261	31	10	302	2082	2394	20	0·8354

The terminal values of the last three columns express the relative numbers of living, dying, and rate of Mortality; and will be found to correspond with those assigned to Age 21, in the Table, p.9.

It will be seen that the numbers relating to the Amicable, are placed separately from the rest, this arises from the fractional ages, on admission, being retained as the foundation of the subsequent ages thoughout the Amicable Experience; consequently to obtain results for integer ages, it becomes necessary to adopt the "mean" results between two fractional ages. Thus the results for age 21, (complete,) are taken from the mean results between $20\frac{1}{2}$ and $21\frac{1}{2}$, or the sum of such results as double the results for Age 21, (complete.) In the above Example, the results summed are those appertaining to the Ages $20\frac{1}{2}$ at admission, and $21\frac{1}{2}$ twelve months after admission, such results appertaining to the same Lives and therefore appearing more suitable in present instance for affording a "mean" result, than the admissions at two contiguous ages relating to different persons. The values are taken from Table I, of the Society's Experience.

EXAMPLE,—IN RELATION TO TABLE B, SHOWING THE METHOD OF VERIFYING THE ASSIGNED DATA BY REFERENCE TO THE RELATIVE "EXPERIENCE."

Classes.	I. Years elapsed from Dec 31, after admission.	II. Assurances comprised.	III. Assurances Discontinued.	IV. Assurances Current.	V. Assurances becoming Claims.	VI. Total not passing on.	VII. 2(II-VI).	VIII. V+VI+VII +Eq: IV.	IX. 2(V).	X. Mortality ℘ 1000.
A(1)+A(4)	,, ,, ,,	17545	120	1398	63	1581	,,	,,	,,	,,
A(2)+A(5)	,, ,, ,,	13835	49	1078	45	1172	,,	,,	,,	,,
A(3)+A(6)	,, ,, ,,	9236	35	418	69	522	,,	,,	,,	,,
Equitable	,, ,, ,,	21398	0	0	0	0	,,	,,	,,	,,
Totals	.....	62014	204	2894	177	3275	,,	,,	,,	,,
A(1)+A(4)	Dec. 31. 0	15969	1601	1120	161	2882	,,	,,	,,	,,
A(2)+A(5)	,, ,, ,,	12663	1053	973	141	2167	,,	,,	,,	,,
A(3)+A(6)	,, ,, ,,	8714	961	460	169	1590	,,	,,	,,	,,
Equitable	,, ,, ,,	21398	32	0	105	137	,,	,,	,,	,,
Totals	.....	58739	3647	2553	576	6776	103926	111278	1152	10·35 2
A(1)+A(4)	Dec. 31. 1	13082	202	1093	157	2152	,,	,,	,,	,,
A(2)+A(5)	,, ,, ,,	10496	616	976	150	1742	,,	,,	,,	,,
A(3)+A(6)	,, ,, ,,	7124	527	402	178	1114	,,	,,	,,	,,
Equitable	,, ,, ,,	21261	1043	172	245	1460	,,	,,	,,	,,
Totals	.....	51963	3088	2650	730	6468	90990	98360	1460	14·843

The values of the last three columns express the relative numbers of living, dying, and rate of mortality; and will be found to correspond with those assigned to similar periods from admission, in Table B, p. viii.

The above Example is not only useful as one of verification, but also shows the importance of not deducing *general* conclusions from the Experience of a *single* society however extensive; thus, while the *general* ratios of mortality of the first and second years after admission, compared with each other are as 1 to 1·4, those deduced from the Equitable Experience, though numerically less, would have been as 1 to 2·4; a ratio, between two successive years, not likely to recur generally, either in the Equitable or any other Society. The Amicable Society's Experience being less extensive than that of the Equitable, and being moreover subdivided as to admissions into two periods, the one terminating, the other commencing at or about April 5th, 1808, (the date at which the Ages of applicants were first systematically demanded by the Amicable,) the resulting data thus subdivided, are not sufficiently numerous to allow of being continuously and usefully traced, year by year, for 25 years, in the same manner as the remaining data constituting Table B, and therefore are not included in that Table; the more especially as the adoption of fractional ages from admission, already alluded to in preceding exemplification of Table A, would require annual adjustments throughout the whole period.

The verification of the assigned data upon which Table C is founded can be effected by simple inspection of those set forth in the Table denoted by F in the original Experience. A similar process will test the validity of the methods of calculation advocated in the present work, by comparing the Expectation or average duration of life, arising from the original probabilities of living, as set forth in such Table, in connection with the Expectation deduced from the same data constituting Table CC p. xiv., in which, for practical purposes, the rate of mortality has been made to annually augment.

To facilitate this latter comparison, the original Expectation, or that of Table F is annexed, as calculated by the Committee of Actuaries, and a Table denoted in the same Experience by G, and calculated by the same Committee, is also presented, to further illustrate the importance, already alluded to, in preceding exemplification of Table B, of not deducing general conclusions from particular classes of data. For Table F being founded by the Committee upon the Total Experience or 83905 Assurances, including 9236 Irish and 12132 Assurances not capable of classification; and Table G being founded by the same Committee upon 62537 Assurances from the Combined Town and Country Experience, but excluding the Irish and other Assurances before stated, it might be hastily conjectured that the two Tables would materially differ, considering the increased rates of Mortality displayed in the relative Table D (5) professedly representing the Irish Experience. The comparison will show however that the two Tables are as nearly allied as any two others that can be named, not wholly founded upon the same data.

The similarity of Tables F and G, notwithstanding the increased rates of Mortality in the Table marked D (5), in the original Experience, can only be accounted for, by presuming the Returns constituting D (5) to be incomplete; a circumstance probably arising from nearly all the Irish deaths being specified in the Returns, while an undue proportion of the relative survivors were left undetermined among the indiscriminate Assurances; the total number of Assurances relating to Ireland being difficult of exact ascertainment. Should this supposition be correct, it is obvious that no trustworthy conclusions can be deduced from Table D, (5) alone, as to the real rates of Mortality prevailing among Irish Lives, although the data may be usefully combined with the other Returns in producing a result more typical of *general* Assurances than any selection of data could be supposed to represent.

It is not however to be inferred that the Tables F and G, or indeed any other actuarial Tables, are intended to show that Agency transactions in Ireland, or elsewhere, can be as safely negociated as those carried on under the immediate superintendance of Principals, but it is the vocation of the Calculator to test opinions by the results appertaining to them, and thus to show, in this sense at least, which are and which are not to be rated as opinions of an extreme character.

Comparison of the Average Duration or Expectation of Life as denoted by Two Tables, F and G, Calculated by the Committee of Actuaries alreadv referred to.

TABLE F.

Age.	Total Experience. 83905 Assurances *Including* Irish, &c.
21	40·45
22	39·92
23	39·18
24	38·54
25	37·84
26	37·13
27	36·42
28	35·76
29	35·06
30	34·25
31	33·50
32	32·75
33	31·98
34	31·27
35	30·55
36	29·90
37	29·20
38	28·51
39	27·79
40	27·07
41	26·41
42	25·68
43	24·98
44	24·26
45	23·55
46	22·85
47	22·12
48	21·41
49	20·79
50	20·11
51	19·46
52	18·79
53	18·16
54	17·50
55	16·83
56	16·23
57	15·62
58	14·98
59	14·38
60	13·81
61	13·24
62	12·68
63	12·09
64	11·50
65	11·03
66	10·51
67	10·03
68	9·46
69	8·99
70	8·50

TABLE G.

Age.	Town and Country Experience. 62537 Assurances *excluding* Irish, &c.
21	40·79
22	40·09
23	39·39
24	38·68
25	37·98
26	37·27
27	36·56
28	35·86
29	35·15
30	34·43
31	33·72
32	33·01
33	32·30
34	31·58
35	30·87
36	30·15
37	29·44
38	28·72
39	28·00
40	27·28
41	26·56
42	25·84
43	25·12
44	24·40
45	23·69
46	22·97
47	22·27
48	21·56
49	20·87
50	20·18
51	19·50
52	18·82
53	18·16
54	17·50
55	16·86
56	16·22
57	15·59
58	14·97
59	14·37
60	13·77
61	13·18
62	12·61
63	12·05
64	11·51
65	10·97
66	10·46
67	9·96
68	9·47
69	9·00
70	8·54

The results of Table F, or those founded on them, as in Tables C, CC, pp. xiii. xiv., also show that such results are equivalent, in a practical sense, to the mixture of Tables A, and B, pp. vi. viii., or of AA, and BP, p. xiv. as will be seen in the Comparative Tables, p. xiii. It was this equivalence that induced the author to found Table C upon the total data, or 83905 Assurances, in place of excluding the indiscriminate Assurances, by which process, the total number would have been proportionately reduced; the reduction being matter of necessity in constructing the Tables A and B, as is evident from the nature of the details they are intended to illustrate. Some callators might have been induced to exclude the Irish Assurances, but such an exclusion would have restricted the general application of the Tables, and has already, probably prevented more than one Society adopting Table G, from its appearing as an exclusive Table, although as shown above, the results are closely approximate to those of Table F, in framing which no such exclusion was adopted.

As a conclusion to the present Part, the general observation may be made;—that Life Assurance Societies occasionally err in both extremes as to the real value of selection, it being a matter of common occurrence for Lives refused admission, as not select, by one Society, to be freely admitted, as select, by another; indeed it is a fair supposition that there is no Society existing, or perhaps capable of being instituted, that would not find upon comparison that certain Lives had been accepted as select, which had been or would have been declined, as not select, by other Societies not professing more than ordinary care in their selections. If then reliance may be placed upon the supposition, that Lives selected under the prevailing system eventually represent but an average of good and bad selections, a species of verbal interpretation is afforded to the numerical results deduced from Table B and the corroborative opinions of Mr. Milne already quoted in connection with it, to the effect, that the ultimate Experience of Life Assurance Societies, excluding the temporary effect of new admissions, would not materially differ as to results from those deducible from Observations on the Population at large without discrimination as to health, *vide p.* xi.

In further illustration of such deductions, and as a topic of general interest, it may be useful and satisfactory to quote, in addition to Mr. Milne's, the opinion of the equally able Mathematician and Actuary, Mr. Gompertz; as in the subjoined note, which is extracted from the evidence of Mr. Gompertz before a Parliamentary Committee on Friendly Societies in 1827.*

* The three following extracts are arranged in the order most suitable to the deductions contained in them; the questions, producing the replies, being almost necessarily in desultory order when originating from different members of a Committee.

(To Mr. Gompertz).—"Have you any observation to make upon any of the subjects before the Committee?—I think it would be serviceable to get all the information which can be obtained from the long established Insurance Societies, with respect to the number of deaths at certain ages, which have been of a certain different number of years standing in the Society, by that means we shall have an opportunity of judging whether there is, as some suppose, a great difference between selected and unselected lives; *I apprehend the difference will not be great*, (Parl. Rep. p. 33.)

"Do you not conceive that the circumstance of the members of the Equitable Society consisting of selected persons makes a difference?—I do not think it makes so much difference as is generally supposed; my reason for this opinion is, that when the comparison is taken from the actual receipts, perhaps it will be found, that if persons in a bad state of health were to be admitted into the Society, in a few years they would either recover from their bad health, or be dead, and that they would be taken off the books, and therefore it would make but very little difference in future estimations. It appears to me, that if a certain number of persons are taken by an Insurance office, and the selection of Life should not happen to be made with what may be termed sufficient judgment, that although the Society would in a few years experience a loss greater than it expected, still in the long run it would make no great difference in the actual annual receipts, when the Society's books are compared with the books of Selected Lives, and for this reason, that although there might have been some bad Lives originally, yet when a comparison is made of the annual premiums, say ten years after, that those Lives if actually so bad that they could not recover, would be dead, and therefore the Society would be continually receiving annual premiums of Lives in very similar circumstances with the Selected Lives. (Parl. Rep. p. 29.)

"Do you think that a Table formed from that Society's Experience gives the true duration of Life among mankind in general?—It appears to me not exactly so, but when Lives are taken off it, and they are careful not to receive imposition, *I think there would be very little difference;* for the number of bad Lives whose state of health is such as not to be re-established, and still not too bad to take, is very small. (Parl. Rep. p. 29.)

A comparison of the above opinions with those cited as Mr. Milne's in p. xi. will show their similarity.

The coincidence of opinion of two of the most eminent Mathematicians who have been professionally engaged as Actuaries, and the confirmation of those opinions by the results of Table B, leave scarcely any doubt, as to the value of selection, (considering how it has to be exercised,) being a much less boon to Life Assurance Societies than has been commonly supposed; its effect being merely of a temporary character and eventually limited, to the exclusion of the worst descriptions of Lives, as a counterbalance to the absence of numerous healthy Lives among the Population at large, who do not become included as Assured Lives.*

* At present date, (Jan. 1850,) there are no materials extant which enable us to judge accurately of the value of *self-selection* as in the case of certain classes of Annuitants, but if the results of Twenty-years additional Experience among the Government Life-Annuitants,—results which Mr. Finlaison, the Government Actuary, recently informed the present author, were then in course of preparation,—should confirm certain indications deducible from Mr. Finlaison's previous "Observations," there will be sufficient grounds for asserting that a number of originally self-elected Annuitants, especially Female Annuitants, if annually nominated by *lot* from among such Annuitants generally, would not be found to experience a greater amount of mortality than would prevail among a similar number of Lives annually nominated by *selection* from communities of persons seeking Life-Assurance: not however as in the case of Annuities to provide for longevity, but as a pecuniary provision against the evils of premature death.

ERRATA. For 6·2263, at Age 69, in Table P, p. vi., read 6·2163.
For 48·8662, at Age 70, in Table P, pp. vi. vii., read 48·8562.
For 107·9554, at Age 70, in Table, at p. vii., read 107·9454.